Problem set 8

Oleksiy Yevdokimov
University of Southern Queensland
<oleksiy.yevdokimov@usq.edu.au>

The purpose of the section is to supply teachers and students with a selection of interesting problems. In this issue we invite readers to look back as far as more than one hundred years ago and work on a selection of problems that had been proposed to students and teachers at that time. The tradition of publishing problems in periodicals was well established in Europe by the beginning of the nineteenth century. The first two problems appeared in *The Mathematical Visitor* (Martin, 1881) that was published in the seventies of the nineteenth century in the US. The next two problems are from the Russian periodical *Vestnik (Newsletter) of the Experimental Physics and Elementary Mathematics* (Tsimmerman, 1886–1917). The final problem comes from the Italian mathematical magazine *Il Pitagora* where it appeared in 1897.

1. Solve the system of equations $x^2 + y = 11$ and $x + y^2 = 7$.

2. Compute the number indicated by 4^{4^4}.

3. Solve the system $\dfrac{y(1-xy)}{1+y^2} = m$ and $\dfrac{x(1-xy)}{1+x^2} = n$.

4. Show that $n^6 - 3n^5 + 6n^4 - 7n^3 + 5n^2 - 2n$ is divisible by 24 for any positive integer n.

5. Let a and b be numbers non-divisible by 7. Prove that $a^6 - b^6$ is divisible by 7.

References

Author Unknown (1897). *Il Pitagora: Giornale di matematica per gli alunni delle scuole secondarie* (1895–1918) (in Italian).

Martin, A. (Ed.) (1881). *The mathematical visitor* (Vol. 1, 1877–1881). Erie, PA: Author. Available: http://www.archive.org/stream/mathematicalvis00martgoog#page/n34/mode/1up

Tsimmerman, V. (Ed). (1886-1917). *Vestnik (newsletter) of the Experimental Physics and Elementary Mathematics*. Available online at http://www.vofem.ru (in Russian, republished by the Russian Academy of Sciences, Institute of Mathematics.)

Solutions to this set of problems for publication should be submitted to:
Oleksiy Yevdokimov, Department of Mathematics and Computing,
University of Southern Queensland, Baker Street, Toowoomba, QLD 4350
or by email to yevdokim@usq.edu.au.
Solutions to this set will be made available on the AAMT website (www.aamt.edu.au) after 1 May 2012.